We wish to find an orthonormal basis for our vector space. Let

$$\mathscr{S} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

Note that this is a spanning set for $\mathbb{R}^3$. We begin by obtaining an orthogonal basis which spans $\mathbb{R}^3$:

$$b_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{b_1 \cdot b_2}{\|b_1\|^2} b_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{\begin{pmatrix} 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}}{2} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$b_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \frac{b_2 \cdot b_3}{\|b_2\|^2} b_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \frac{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -1/3 \\ 2/3 \end{pmatrix}$$